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Free Convection Effects on Stokes Problem for An Infinite Vertical Plate Using Finite Element Technique

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***Abstract*—A finite element technique is implemented for the free convection effects on stokes problem for an infinite vertical plate. The numerical values for velocity and temperature under various physical situations are obtained and have been analyzed graphically. The numerical results are compared with analytical solution and established the validity of finite element technique (Ritz method).**

***Index Terms*—specific heat, Grash of number, velocity, kinematic Viscosity, Finite element method.**

I. INTRODUCTION

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane was studied first by Stokes [10]. It is known as Stokes problem in the literature. Because of its practical importance, it has been extended to bodies of different shapes by a number of researchers. Amongst them are Illingworth [5, Stewartson [9], Hall [4] and Elliott [3]. Illingworth considered the flow of a compressible gas with variable viscosity near an impulsively started vertical plate and the problem was solved by the method of successive approximation. Stewartson studied the flow of a viscous incompressible fluid past an impulsively started semi-infinite horizontal plate. Hall considered the flow past an impulsively started semi-infinite horizontal plate by finite difference method of mixed explicit-implicit type, which is convergent and stable hence it is free from any restrictions on the mesh size.

However, in Hall's analysis, only unsteady velocity field was studied. Elliott generalized Illingworth's problem by assuming a time dependent velocity and temperature for the plate, but neglected the viscous dissipative heat. However, in the above paper, only mathematical results were derived and no physical situation was discussed. In fact in such type of problems, from the engineering point of view, the physical aspects are very important.

So in order to present the physical situation of the flow past an impulsively started infinite vertical plate, Soundalgekar[12] considered the problem for incompressible fluid with constant

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properties. Soundalgekar[12] was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite vertical plate in its own plane. In this paper, the effect of maintaining the plate at different uniform temperature T_w was studied. Now, in this type of flow we can consider two physical situations. (1) The temperature of the plate is the same as that of the fluid extended to infinity; (2) the plate temperature differs from the temperature of the fluid at infinity. The second case is more important from both the physical and practical point of view. If the difference between the plate temperature T_w' and that of the fluid at infinity T_∞' .viz. $T_w' - T_\infty'$ is appreciable, then free convection currents are induced in the vicinity of the plate. Here is an attempt to study such a physical situation using Finite Element Method and hence, it is presented here. This problem has a variety physical application, example, Filtration process, the drying of porous materials in textile industries, etc. However, in many applications in space technology, the plate temperature doesn't remain uniform but varies with time. How the flow field will be affected by such a physical phenomenon? In order to clear this, Soundalgekar and Patil studied the Stokes problem for an infinite plate, whose temperature varies linearly with time and given a conclusion that the velocity near the plate increases when the time t and the Grashof number G_r increases.

It is well known fact that, the transition from conduction to convection is only due to the leading-edge effect that has propagated to a certain point. The distance of this point from the leading edge is known as maximum penetration distance $X_{Pmax.}$ Also, X_P is a function of t_1 (time) and y_1 (coordinates normal to the plate). Stokes also studied the flow of a viscous fluid past an infinite horizontal plate oscillating in its own plane. It has been proposed to study the effects of oscillating vertical plate and free convection currents on the flow of a viscous fluid which is stationary and extending to infinity. As the leading-edge effects are important in such a problem, it is interesting to know the effects of plate oscillations and the free convection currents on the leading-edge effects. So Soundalgekar studied the free convection effects on the flow past an infinite vertical oscillating plate by taking into account the presence of free convection currents. This study has been found to be useful in the design of space ships.

Mathematical Formulation

Here x_1 - axis is taken along an infinite plate in the vertical direction and y_1- axis is taken normal to the plate. Initially, the temperature of the plate is the same as that of the fluid. At $t > 0$ the plate starts moving impulsively in its own plane with velocity U_{0B} and its temperature is instantaneously raised or lowered to T_P which is thereafter maintained constant.

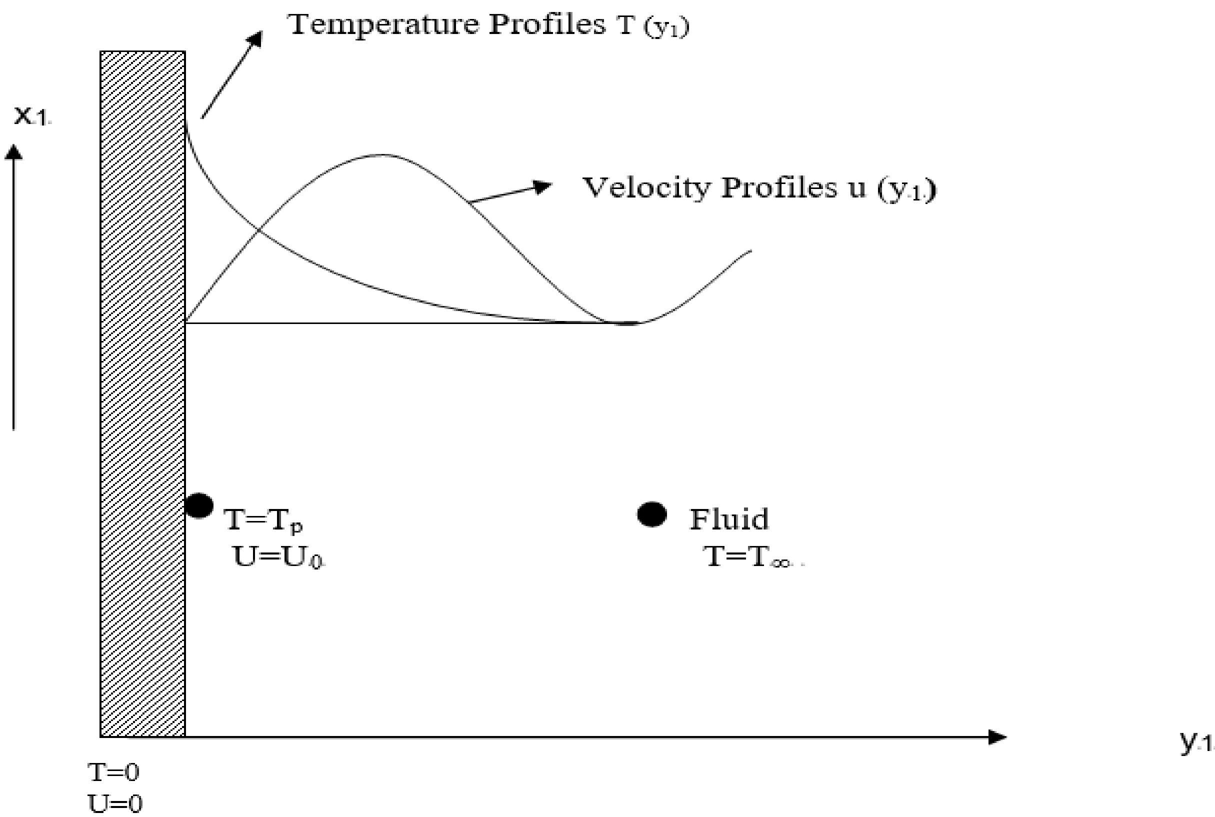

Fig 1: Geometry of the problem

As the plate is infinite, all variables in the problem are functions of y_1 and t_1 only. Therefore, the components of velocity are given by

$$v_{x_1} = u_1(y_{1,}t_1) \; ; \; v_{y_1} = 0 \;\text{ and }\; v_{z_1} = 0. \tag{1}$$

All fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. Thus, the problem is governed by the following equations

$$\frac{\partial u_1}{\partial t_1} = g\beta(T_1 - T_\infty) + \nu \frac{\partial^2 u_1}{\partial y_1^2}, \tag{2}$$

$$\rho_1 c_p \frac{\partial T_1}{\partial t_1} = k \frac{\partial^2 T_1}{\partial y_1^2}. \tag{3}$$

Here, u_1 is the velocity of the fluid, T_1 the temperature of the fluid near the plate, T_∞ the temperature of the fluid far away from the plate, g the acceleration due to gravity, β the coefficient of volume expansion, ν the kinematic viscosity, ρ_1 the density of the fluid, k the thermal conductivity, c_p specific heat at constant pressure and t_1 is the time. In equation (3) the heat due to viscous dissipation is assumed to be negligible. This is possible when the velocity is small.

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The boundary conditions are

$$\left.\begin{aligned} & u_1(y_1, t_1) = U_0, \\ & T_1(y_1, t_1) = T_p \qquad \text{at } y_1 = 0; \\ & u_1(y_1, t_1) \to 0, \\ & T_1(y_1, t_1) \to T_\infty \qquad \text{as } y_1 \to \infty. \end{aligned}\right\} \tag{4}$$

On introducing the following non-dimensional quantities

$$\left. y = \frac{y_1 U_0}{\nu}, \right\}$$

$$\left.\begin{aligned} & t = \frac{t_1 U_0^2}{\nu}, \\ & u = \frac{u_1}{U_0}, \\ & T = \frac{T_1 - T_\infty}{T_p - T_\infty}, \\ & P_r = \frac{\mu c_p}{k} \ (\text{Prandtl number}), \\ & G_r = \frac{\nu g \beta (T_P - T_\infty)}{U_0^3} \ (\text{Grashof number}), \end{aligned}\right\} \tag{5}$$

in equations (2), (3) and (4) we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r T, \tag{6}$$

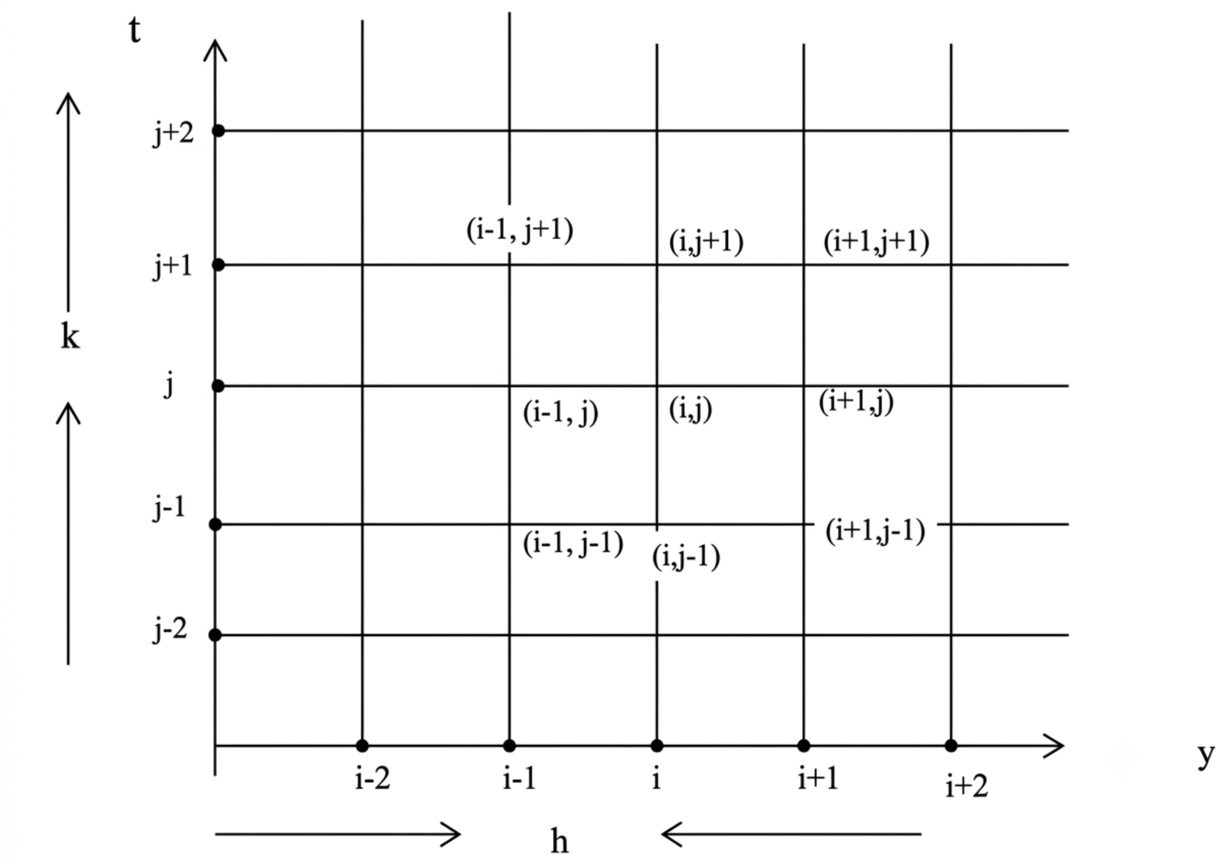
$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2}. \tag{7}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\left.\begin{aligned} & t < 0, \quad u(y, t) = 0, \quad T(y, t) = 0; \\ & t > 0, \quad u(0, t) = 1, \quad T(0, t) = 1; \\ & t > 0, \quad u(\infty, t) = 0, \quad T(\infty, t) = 0. \end{aligned}\right\} \tag{8}$$

II. METHOD OF SOLUTION

The Finite Element Method (i.e., Ritz method) has been used to solve equations (6) and (7) subject to the boundary condition (8). To obtain the difference equations, the region is divided into a grid or mesh of lines parallel to y and t axis. Solution of the difference equations is obtained at the intersection of these mesh lines, called nodes. The values of the dependent variables u and T at the nodal points along the planes y=0 and t=0 are given by u (0, t), T (0, t), u (y, 0) and T(y, 0) thus are known from the initial and boundary conditions. In the Figure (2) h, k are the mesh sizes along y and t directions respectively. A scheme is required to find single value at next time level in terms of known values from present time level. We use finite element method and apply trapezoidal rule to get the Crank-Nicolson method.



Now, we consider equation (7)

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2}.$$

Using finite element method with crank-nicolson discretization taking h=0.05, k=0.0025 therefore $r = k / h^2 = 1$.

The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem may be written as

$$j^e = 1/2\int_{yj}^{yk} \left[\left(\frac{\partial T^e}{\partial y}\right)^2 + 2T^e p_r \frac{\partial T^e}{\partial t}\right]dy$$

For the linear piecewise approximate solution

$$T^e = N_J(y)T_j(t) + N_k(y)T_k(t)$$

The element equation is given by

$$j^e = 1/2\int_{y_j}^{y_k}\left\{\left([N_j^{'} \quad N_k^{'}]\begin{bmatrix}T_J\\T_k\end{bmatrix}\right)^2 + 2P_r[N_J \quad N_K]\begin{bmatrix}T_J\\T_k\end{bmatrix}[N_j \quad N_K]\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial T_j} = 1/2\int_{y_j}^{y_k}\left\{2\left([N_j^{'} \quad N_k^{'}]\begin{bmatrix}T_j\\T_k\end{bmatrix}\right)N_j^{'} + 2P_r N_j[N_j \quad N_k]\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial T_j} = \int_{y_j}^{y_k}\left\{\left([N_j^{'} \quad N_k^{'}]\begin{bmatrix}T_j\\T_k\end{bmatrix}\right)N_j^{'} + P_r N_j[N_j \quad N_k]\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial T_k} = \frac{1}{2}\int_{y_j}^{y_k}\left\{2\left([N_j^{'} \quad N_k^{'}]\begin{bmatrix}T_j\\T_k\end{bmatrix}\right)N_k^{'} + 2P_r N_k[N_j \quad N_k]\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial T_k} = \int_{y_j}^{y_k}\left\{\left([N_j^{'} \quad N_k^{'}]\begin{bmatrix}T_j\\T_k\end{bmatrix}\right)N_k^{'} + P_r N_k[N_j \quad N_k]\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k}\left\{\begin{pmatrix}N_j^{'}[N_j^{'} & N_k^{'}]\\N_k^{'}[N_j^{'} & N_k^{'}\end{pmatrix}\begin{bmatrix}T_j\\T_k\end{bmatrix} + P_r\begin{pmatrix}N_j[N_j & N_k]\\N_k[N_j & N_k]\end{pmatrix}\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k}\left\{\begin{pmatrix}[N_j^{'}N_j^{'} & N_j^{'}N_k^{'}]\\ [N_k^{'}N_j^{'} & N_k^{'}N_k^{'}\end{pmatrix}\begin{bmatrix}T_j\\T_k\end{bmatrix} + P_r\begin{pmatrix}[N_jN_j & N_jN_k]\\ [N_kN_j & N_kN_k]\end{pmatrix}\begin{bmatrix}\dot{T}_j\\\dot{T}_k\end{bmatrix}\right\}dy$$

Where prime denotes differentiation w.r.t. to ‘y’ and dot represent differentiation w.r.t. to ‘t’

Here $N_j^{'} = \frac{-1}{h}$; $N_k^{'} = \frac{1}{h}$; where h= $y_k - y_j$;

Simplifying, we get

$$\frac{1}{h}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}T_J\\T_K\end{bmatrix} + \frac{h}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}\dot{T}_J\\\dot{T}_K\end{bmatrix} = 0$$

The condition for the extremization of the functional j^e w.r.t the nodal values T_j and T_k are given by

$$\frac{\partial j^e}{\partial \phi^e}=\sum_{e=1}^{n+1}\frac{\partial j^e}{\partial \phi^e}=0$$

In order to get the differential equation at the knot y_i we write the element equations for the elements

$$y_{i-4}\le y\le y_{i-3};\ y_{i-3}\le y\le y_{i-2};\ y_{i-2}\le y\le y_{i-1};\ y_{i-1}\le y\le y_i;$$
$$y_i\le y\le y_{i+1};\ y_{i+1}\le y\le y_{i+2};\ y_{i+2}\le y\le y_{i+3};y_{i+3}\le y\le y_{i+4};$$

For the elements $y_{i-4}\le y\le y_{i-3}$ and $y_{i-3}\le y\le y_{i-2}$

the assembled equation is given as

$$\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 2+2 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{T}_{i-4}\\ \dot{T}_{i-3}\\ \dot{T}_{i-2}\end{bmatrix}+\frac{1}{p_r h^2}\begin{bmatrix}1 & -1 & 0\\ -1 & 1+1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}T_{i-4}\\ T_{i-3}\\ T_{i-2}\end{bmatrix}=0$$

Thus, assembling all the elements we get

$$\begin{bmatrix}2&1&0&0&0&0&0&0&0\\1&4&1&0&0&0&0&0&0\\0&1&4&1&0&0&0&0&0\\0&0&1&4&1&0&0&0&0\\0&0&0&1&4&1&0&0&0\\0&0&0&0&1&4&1&0&0\\0&0&0&0&0&1&4&1&0\\0&0&0&0&0&0&1&4&1\\0&0&0&0&0&0&0&1&4\end{bmatrix}\begin{bmatrix}\dot{T}_{i-4}\\ \dot{T}_{i-3}\\ \dot{T}_{i-2}\\ \dot{T}_{i-1}\\ \dot{T}_{i}\\ \dot{T}_{i+1}\\ \dot{T}_{i+2}\\ \dot{T}_{i+3}\\ \dot{T}_{i+4}\end{bmatrix}+$$

$$\frac{1}{p_r h^2}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} T_{i-4} \\ T_{i-3} \\ T_{i-2} \\ T_{i-1} \\ T_i \\ T_{i+1} \\ T_{i+2} \\ T_{i+3} \\ T_{i+4} \end{bmatrix} = 0$$

We put the row corresponding to the knot ‘i’ to zero and write the semi discretization as

$$\frac{1}{6}\left(\dot{T}_{i-1} + 4\dot{T}_i + \dot{T}_{i+1}\right) = \left(\frac{1}{p_r h^2}\right)\left(T_{i-1} - 2T_i + T_{i+1}\right)$$

Applying the trapezoidal rule we obtain the crank-nicolson method

$$\frac{1}{6k}\left[\left(T_{i-1}^{n+1} - T_{i-1}^{n}\right) + 4\left(T_i^{n+1} - T_i^{n}\right) + \left(T_{i+1}^{n+1} - T_{i+1}^{n}\right)\right] =$$

$$\frac{1}{p_r h^2}\left[\frac{\left(T_{i-1}^{n+1} + T_{i-1}^{n}\right)}{2} - 2\frac{\left(T_i^{n+1} + T_i^{n}\right)}{2} + \frac{\left(T_{i+1}^{n+1} + T_{i+1}^{n}\right)}{2}\right]$$

$$\left[\left(T_{i-1}^{n+1} - T_{i-1}^{n}\right) + 4\left(T_i^{n+1} - T_i^{n}\right) + \left(T_{i+1}^{n+1} - T_{i+1}^{n}\right)\right] =$$

$$\frac{3r}{p_r}\left[\frac{\left(T_{i-1}^{n+1} + T_{i-1}^{n}\right)}{2} - 2\frac{\left(T_i^{n+1} + T_i^{n}\right)}{2} + \frac{\left(T_{i+1}^{n+1} + T_{i+1}^{n}\right)}{2}\right]$$

Simplifying above equation we get

$$\left(1 - \frac{3r}{p_r}\right)T_{i-1}^{n+1} + \left(4 + \frac{6r}{p_r}\right)T_i^{n+1} + \left(1 - \frac{3r}{p_r}\right)T_{i+1}^{n+1} =$$

$$\left(1 + \frac{3r}{p_r}\right)T_{i-1}^{n} + \left(4 - \frac{6r}{p_r}\right)T_{i-1}^{n} + \left(1 + \frac{3r}{p_r}\right)T_{i-1}^{n}$$

For h=0.05, k=0.0025 and r =1 the nodal points (y_n, t_n) are shown in the figure (2), where i =1(1)9 and n=1, 2, 3….

We solve the system of above equations using Gauss –Seidel method.

t	Y	Analytical Solution	Numerical Solution	Percentage of Error
0.0075	0	1	1	0.0000
0.0075	0.05	0.7261	0.6799	0.0005
0.0075	0.1	0.4846	0.4664	0.0002
0.0075	0.15	0.2944	0.2502	0.0004
0.0075	0.2	0.1621	0.1144	0.0005
0.0075	0.25	0.0805	0.0472	0.0003
0.0075	0.3	0.0360	0.0182	0.0002
0.0075	0.35	0.0144	0.0067	0.0001
0.0075	0.4	0.0052	0.0023	0.0000
0.0075	0.45	0.0017	0.0008	0.0000
0.0075	0.5	0.0005	0.0003	0.0000
0.0075	0.55	0.0001	0.0001	0.0000
0.0075	0.6	0.0000	0.0000	0.0000
0.0075	0.65	0.0000	0.0000	0.0000
0.0075	0.7	0.0000	0.0000	0.0000
0.0075	0.75	0.0000	0.0000	0.0000
0.0075	0.8	0.0000	0.0000	0.0000
0.0075	0.85	0.0000	0.0000	0.0000
0.0075	0.9	0.0000	0.0000	0.0000
0.0075	0.95	0.0000	0.0000	0.0000
0.0075	1	0.0000	0.0000	0.0000

Table 1: Comparison of temperature profiles when P_r=0.7333

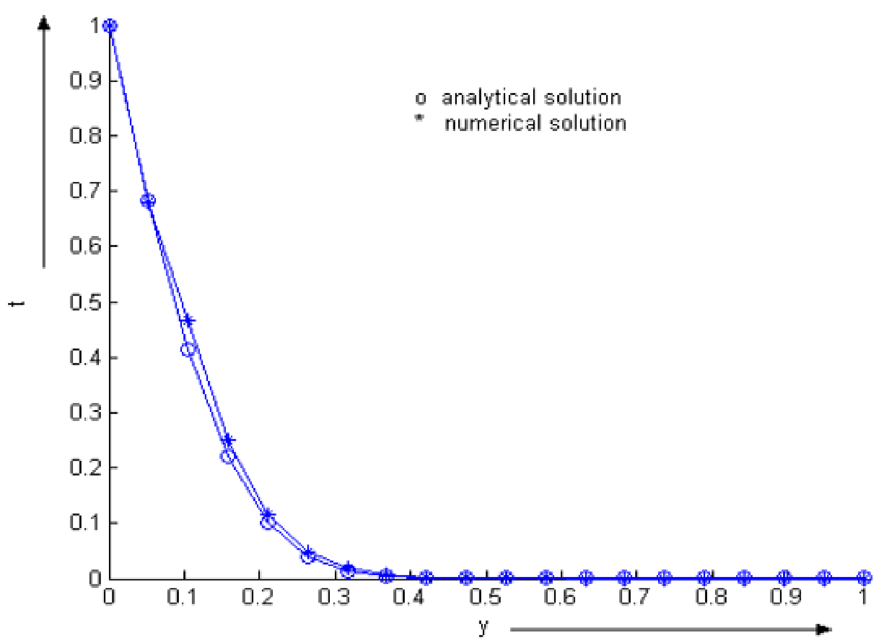


Fig 3: Comparison of temperature Profiles when P_r=0.7333

Discussion:

Comparisons of Numerical Solutions of temperature with analytical solutions (annexure 2) are plotted in figure (3) here the accuracy is very high between the numerical solutions and analytical solutions. Therefore we can say that Finite Element Method is Valid.

Now, we consider equation (6)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r T$$

Using finite element method with crank nicolson- discretization taking h=0.05 and r=1

The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem may be written as

$$j^e = 1/2 \int_{yj}^{yk} \left[\left(\frac{\partial U^e}{\partial y} \right)^2 + 2U^e \frac{\partial U^e}{\partial t} - 2U^e G_r T \right] dy$$

For the linear piecewise approximate solution

$$U^e = N_J(y) U_j(t) + N_k(y) U_k(t)$$

The element equation is given by

$$j^e = 1/2 \int_{y_j}^{y_k} \left\{ \left([N_j' \quad N_k'] \begin{bmatrix} U_j \\ U_k \end{bmatrix} \right)^2 + 2[N_j \quad N_k] \begin{bmatrix} U_J \\ U_K \end{bmatrix} [N_j \quad N_k] \begin{bmatrix} \dot{U}_J \\ \dot{U}_K \end{bmatrix} - 2[N_J \quad N_K] \begin{bmatrix} U_j \\ U_k \end{bmatrix} G_r T \right\} dy$$

Simplifying above equation we get

$$(1\text{-}3r) U_{i-1}^{n+1} + (4+6r)\ U_i^{n+1} + (1\text{-}3r)\ U_{i+1}^{n+1} =$$

$$(1+3r) U_{i-1}^{n} + (4\text{-}6r)\ U_{i-1}^{n} + (1+3r)\ U_{i-1}^{n} + 6\, G_r T_i^n\ k$$

For h=0.05 and r =1 the nodal points (y_n, t_n) are shown in the figure (2.2)

Where i =1(1)9 and n=1, 2, 3….

We solve the system of above equations using Gauss –Seidel method.

t	y	Analytical Solution	Numerical Solution	Percentage of Error |
0.0075	0	1	1	0.0000
0.0075	0.05	0.726713	0.6421	0.0008
0.0075	0.1	0.48455	0.3924	0.0009
0.0075	0.15	0.294406	0.1706	0.0012
0.0075	0.2	0.162111	0.0615	0.0010
0.0075	0.25	0.080549	0.0197	0.0006
0.0075	0.3	0.035993	0.0059	0.0003
0.0075	0.35	0.014425	0.0017	0.0001
0.0075	0.4	0.005174	0.0005	0.0000
0.0075	0.45	0.001658	0.0001	0.0000
0.0075	0.5	0.000474	0.0000	0.0000
0.0075	0.55	0.000121	0.0000	0.0000
0.0075	0.6	2.74E-05	0.0000	0.0000
0.0075	0.65	5.53E-06	0.0000	0.0000
0.0075	0.7	9.93E-07	0.0000	0.0000
0.0075	0.75	1.58E-07	0.0000	0.0000
0.0075	0.8	2.24E-08	0.0000	0.0000
0.0075	0.85	2.82E-09	0.0000	0.0000
0.0075	0.9	3.15E-10	0.0000	0.0000
0.0075	0.95	3.13E-11	0.0000	0.0000
0.0075	1	2.75E-12	0.0000	0.0000

Table 2: Comparison of Velocity when P_r=6.75

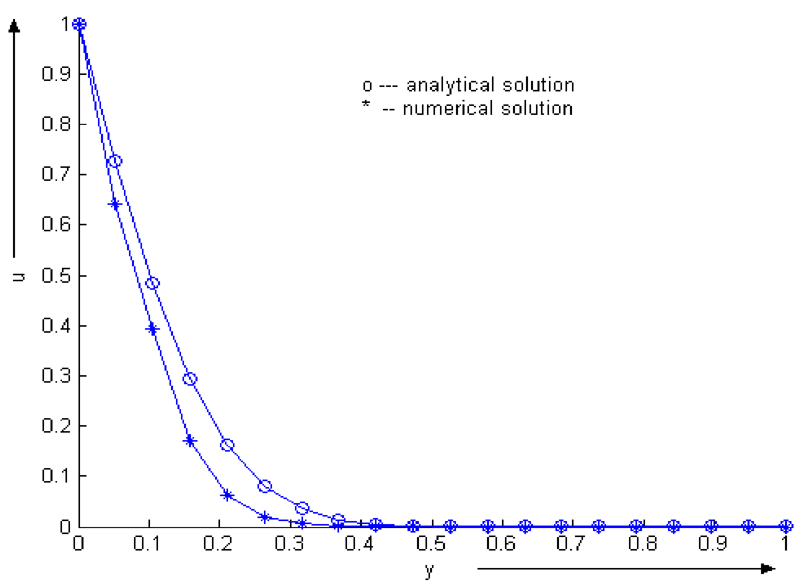


Fig 4: Comparison of Velocity when P_r=6.75

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Discussion

Comparison of numerical solutions of velocity with the corresponding analytical solutions (annexure 4) are plotted in Figure (4). From the figure, it is observed that the error estimation between numerical and analytical values are almost negligible. Thus establishing the validity of the method.

y	GB_{rB} =5 , t=3K	GB_{rB} =5 , t=6K	GB_{rB} =10 , t=3K
0	1.0000	1.0000	1.0000
0.05	0.6480	0.7799	0.6556
0.1	0.4005	0.5617	0.4091
0.15	0.1768	0.3855	0.1829
0.2	0.0648	0.2392	0.0681
0.25	0.0213	0.1387	0.0228
0.3	0.0065	0.0737	0.0071
0.35	0.0019	0.0356	0.0021
0.4	0.0005	0.0157	0.0006
0.45	0.0001	0.0065	0.0002
0.5	0.0000	0.0025	0.0001
0.55	0.0000	0.0009	0.0000
0.6	0.0000	0.0003	0.0000
0.65	0.0000	0.0001	0.0000
0.7	0.0000	0.0000	0.0000
0.75	0.0000	0.0000	0.0000
0.8	0.0000	0.0000	0.0000
0.85	0.0000	0.0000	0.0000
0.9	0.0000	0.0000	0.0000
0.95	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000

Table 3: Numerical values of Velocity when GB_{rB} =5, GB_{rB}=10 and t=3k, t=6k

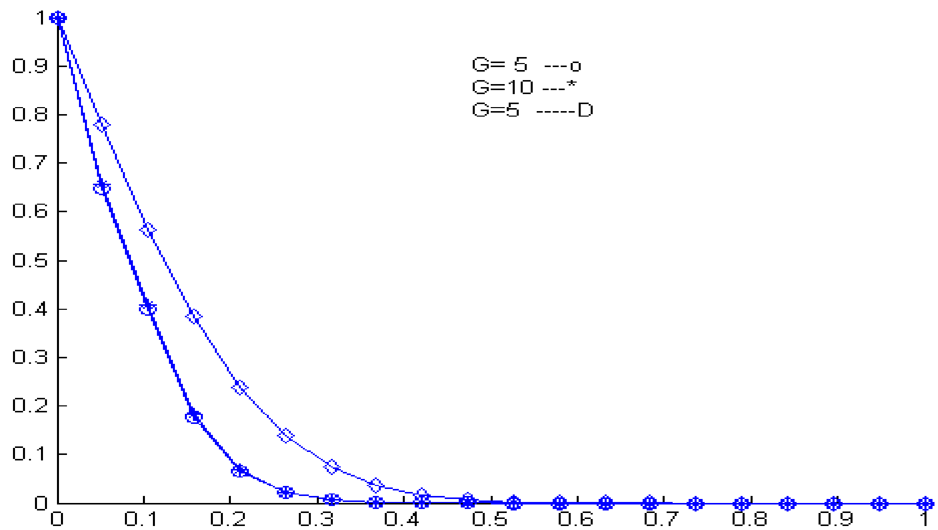

Fig 5: Velocity profiles when GB_{rB} =5, $GB_{r\,B}$=10 and t=3k, t=6k

y	GB_r=-5 , t=3K	GB_r=-5, t=6K	GB_r=-10 , t=3K
0	1.0000	1.0000	1.0000
0.05	0.6330	0.7537	0.6254
0.1	0.3832	0.5272	0.3745
0.15	0.1646	0.3522	0.1585
0.2	0.0582	0.2125	0.0549
0.25	0.0183	0.1197	0.0167
0.3	0.0053	0.0614	0.0046
0.35	0.0014	0.0283	0.0012
0.4	0.0004	0.0118	0.0003
0.45	0.0001	0.0045	0.0001
0.5	0.0000	0.0016	0.0000
0.55	0.0000	0.0005	0.0000
0.6	0.0000	0.0002	0.0000
0.65	0.0000	0.0000	0.0000
0.7	0.0000	0.0000	0.0000
0.75	0.0000	0.0000	0.0000
0.8	0.0000	0.0000	0.0000
0.85	0.0000	0.0000	0.0000
0.9	0.0000	0.0000	0.0000
0.95	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000

Table 4: Numerical values of velocity when GB_{rB} =-5, GB_{rB} =-10 and t=3k, t=6k

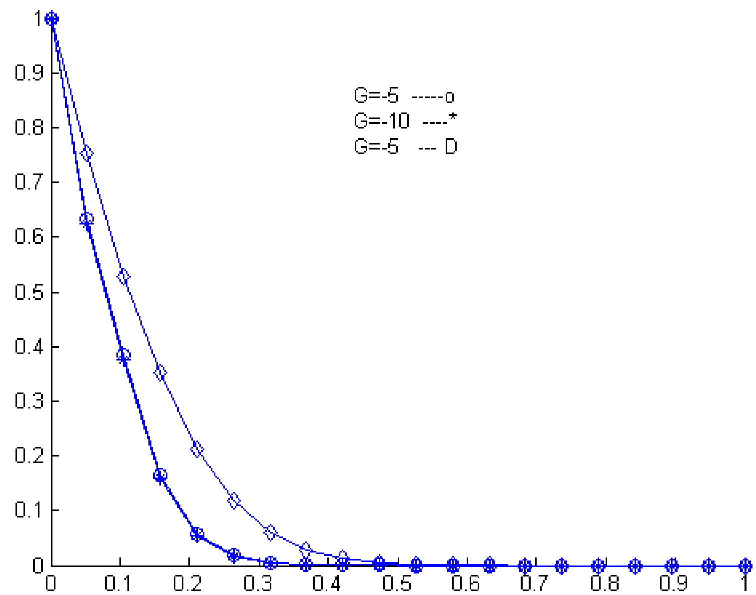

Fig 6: Velocity when G_{rB} =-5, G_{rB}=-10 and t=3k, t=6k

Discussion

The numerical values of velocity profiles for different values of G_r (Grashoff number) and time t have been tabulated under Table (3). From physical point of view $G_r > 0$ corresponds to externally cooled plate. Hence, for an externally cooled plate, we can observe that, as G_r increases, velocity also increase by keeping t fixed which can be seen from figure (5). Also from this figure, it can be observed that as t increases for G_r fixed, the velocity increases. Further, physically $G_r < 0$ corresponds to an externally heated plate as free convection currents are carried towards the plate. For different values of G_{rB} , when time 't' is fixed velocities are computed and presented in table (4) and plotted and shown in figure (6). Further, it is observed that as G_{rB} decreases from -5 to-10, the velocity decreases.

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